SIGNAL TRANSMISSION

Trigonometric, Polar and Complex Fourier Exponential Transforms

ECE 328 – PRINCIPLES OF COMMUNICATION SYSTEMS

Monday, 04 September 2017

ECE328 SYLLABUS

Course Content:

Introduction: definition and importance of communications, History of electronic communication; basic communication block diagram; Electromagnetic spectrum; bandwidth; frequency management. Amplitude modulation: AM spectrum; Modulation Index; Power content; DSB-SC; Hilbert Transform; SSB-SC; Vestigial Sideband Transmission (VSB); Performance comparison of various AM systems. Angle Modulation: Phase Modulation; Frequency modulation (FM); Phasor representation; Wideband FM; FM bandwidth for arbitrary signal; FM generation (Direct method, reactance modulator, Varactor diode method, FET reactance modulator). Demodulating FM waves: classification of FM demodulators; Frequency discriminator; slope detector; Phase discriminator; Performance comparison of FM demodulating methods. AM and FM transmitters and receivers: AM transmitter; SSB transmitter; SSB receivers; FM transmitter; Tuned radio frequency and super heterodyne receivers; Diversity receivers; FM receivers. Noise in communication systems: Noise performance in various modulation schemes, noise figures and signal-to-noise ratio. Communication Signal Transmission: Review of the Fourier transform; signal transmission through a linear system;; signal distortion over a communication channel; signal energy and energy spectral density; signal power and power spectral density; numerical computation of the Fourier transform.

FOURIER SERIES

A **continuous-time Fourier series** exists for a function x(t) when the following conditions (called the Dirichlet's conditions) are satisfied:

1. x(t) is a well-defined single value function
2. x(t) possesses a finite number of discontinuities in the period T
3. x(t) has a finite number of maxima and minima in the period T

TROGONOMETRIC FOURIER SERIES/01

A periodic signal x(t) can be expressed in the form of trigonometric Fourier series comprising the following sine and cosine terms:

$$x(t) = a_0 + a_1 cos\omega_o t + a_2 \cos(2\omega_o t) + a_n \cos(n\omega_o t)$$
$$+ b_1 sin\omega_o t + b_2 \sin(2\omega_o t) + b_n \sin(n\omega_o t)$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(\omega_0 nt) + b_n sin(\omega_0 nt)$$

Where

$$t_o < t < t_0 + T$$

$$T = \frac{2\pi}{\omega_0}$$

ω_0 is the fundamental frequency and $2\omega_o$, $3\omega_o$ are the harmonics

TROGONOMETRIC FOURIER SERIES/02

$$a_o = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt$$

The average value of the DC component of x(t)

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos(\omega_0 nt)\,dt$$

$$= \frac{2}{T}\int_0^T x(t)\cos(\omega_0 nt)\,dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin(\omega_0 nt)\,dt$$

$$= \frac{2}{T}\int_0^T x(t)\sin(\omega_0 nt)\,dt$$

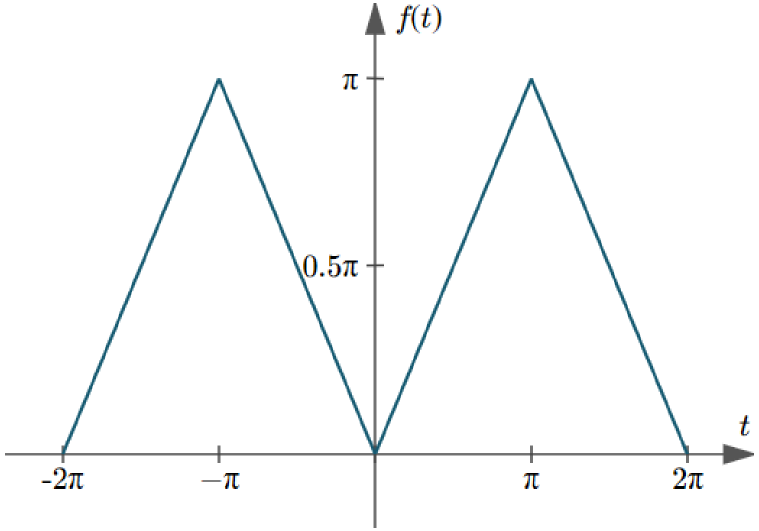
SYMMETRY CONDITIONS FOR INTEGRALS

Even Function

For even function,

x(-t) = x(t)

$$\int_{-a}^{a} x(t)dt = 2\int_{0}^{a} x(t)dt$$



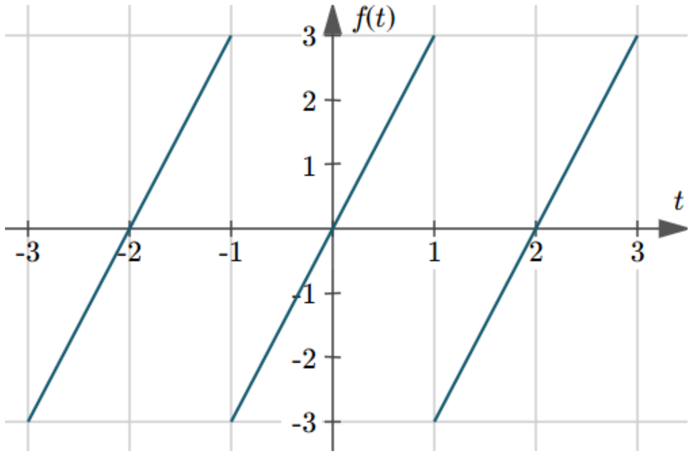
(a) Even triangular function

Odd Function

For odd function,

x(-t) = -x(t)

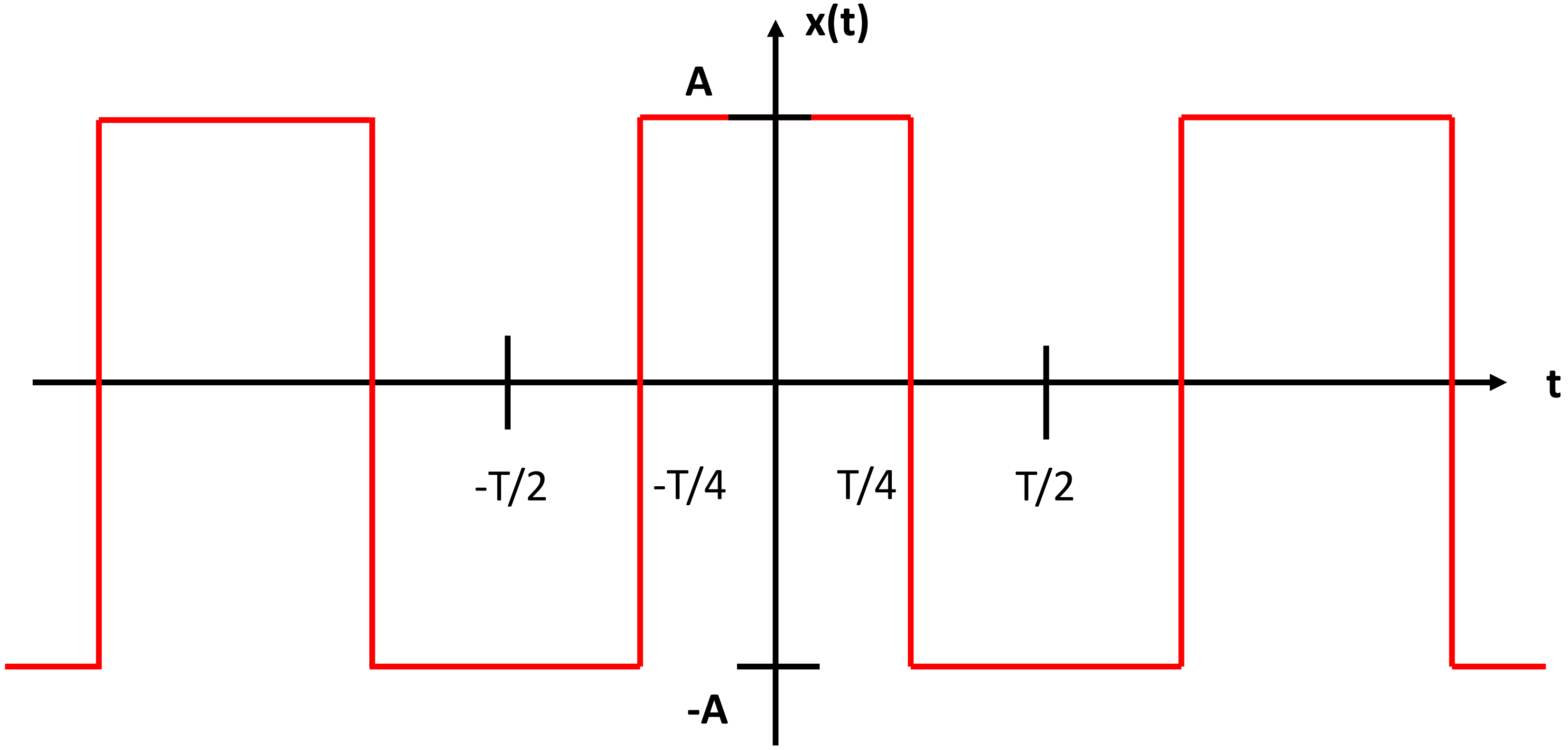
$$\int_{-a}^{a} x(t)dt = 0$$



(b) Odd sawtooth function

WORKED EXAMPLE -1

Obtain the Fourier series representation of a periodic square wave shown below.

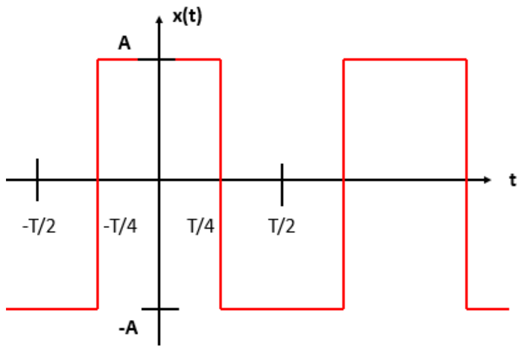


WORKED EXAMPLE -1

The expression for Fourier series is:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(\omega_0 nt) + b_n sin(\omega_0 nt)$$

1. Since the x(t) is symmetrical about the x-axis, i.e x(t) = -x(t), only the cosine terms are present.
2. The waveform is symmetrical about the horizontal axis, therefore the DC term a_0 = 0.



The expected Fourier series should therefore be of the form:

$$x(t) = \sum_{n=1}^{\infty} a_n cos(\omega_0 nt)$$

We can write

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(\omega_0 nt)\,dt = \frac{2}{T}\int_{-T/4}^{T/4} Acos(\omega_0 nt) + \frac{2}{T}\int_{-T/4}^{-T/2} -Acos(\omega_0 nt) + \frac{2}{T}\int_{T/4}^{T/2} -Acos(\omega_0 nt)$$

$$= \frac{8A}{n\omega_0 T} sin\left(\frac{n\omega_0 T}{4}\right) - \frac{4A}{n\omega_0 T} sin\left(\frac{n\omega_0 T}{2}\right)$$

Substituting $\omega_0 = \frac{2\pi}{T}$ the 2nd term becomes zero for all integer values of n, and we remain with

$a_n = \frac{4A}{n\pi} sin\left(\frac{n\pi}{2}\right)$ showing a_n is non-zero for odd values of n

or

$$x(t) = \frac{4A}{\pi}\left(cos\omega_0 t - \frac{1}{3}cos3\omega_0 t + \frac{1}{5}cos5\omega_0 t + \dots\dots\right)$$

POLAR FOURIER SERIES REPRESENTATION

Polar Fourier representation is a modified form of trigonometric Fourier series which we have seen is given by:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(\omega_0 nt) + b_n sin(\omega_0 nt)$$

or

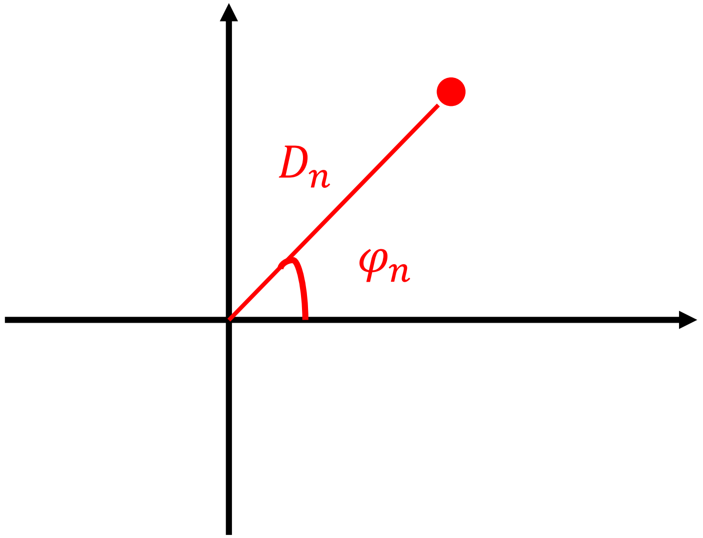
$$x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}\left[\frac{a_n}{\sqrt{a_n^2 + b_n^2}}\cos(\omega_0 nt) + \frac{b_n}{\sqrt{a_n^2 + b_n^2}} sin(\omega_0 nt)\right]$$

If

$\frac{a_n}{\sqrt{a_n^2+b_n^2}} = \cos\varphi_n$ *and* $\frac{b_n}{\sqrt{a_n^2+b_n^2}} = \sin\varphi_n$ So that $\tan\varphi_n = \frac{b_n}{a_n}$

We can then write

$$x(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}(\cos(\omega_0 nt - \varphi_n)$$



where

$D_n = \sqrt{a_n^2 + b_n^2}$ is referred to as spectral amplitude of the function x(t)

φ_n is referred to as the phase spectrum of the function x(t)

COMPLEX FOURIER EXPONENTIAL SERIES/01

The complex for of Fourier series is simpler and more compact. It is therefore more widely used in signal analysis.

From trigonometric Fourier Series,

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(\omega_0 nt) + b_n sin(\omega_0 nt)$$

We know from Euler Identity that

$$e^{i\theta} = cos\theta + jsin\theta \text{ or } e^{-i\theta} = cos\theta - jsin\theta$$

We can write

$$\cos \omega_0 nt = \frac{1}{2}\left(e^{j\omega_o nt} + e^{-j\omega_0 nt}\right)$$

$$\sin \omega_0 nt = \frac{1}{2}\left(e^{j\omega_o t} - e^{-j\omega_0 t}\right)$$

Substituting, we get

$$x(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2}\left(e^{j\omega_o nt} + e^{-j\omega_0 nt}\right) + \frac{b_n}{2}\left(e^{j\omega_o t} - e^{-j\omega_0 t}\right)$$

COMPLEX FOURIER EXPONENTIAL SERIES/01

$$x(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2}\left(e^{j\omega_o nt} + e^{-j\omega_0 nt}\right) + \frac{b_n}{2}\left(e^{j\omega_o t} - e^{-j\omega_0 t}\right)$$

We can write

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{j\omega_o nt} + \sum_{n=-\infty}^{-1} C_{-n} e^{-j\omega_o nt}$$

or

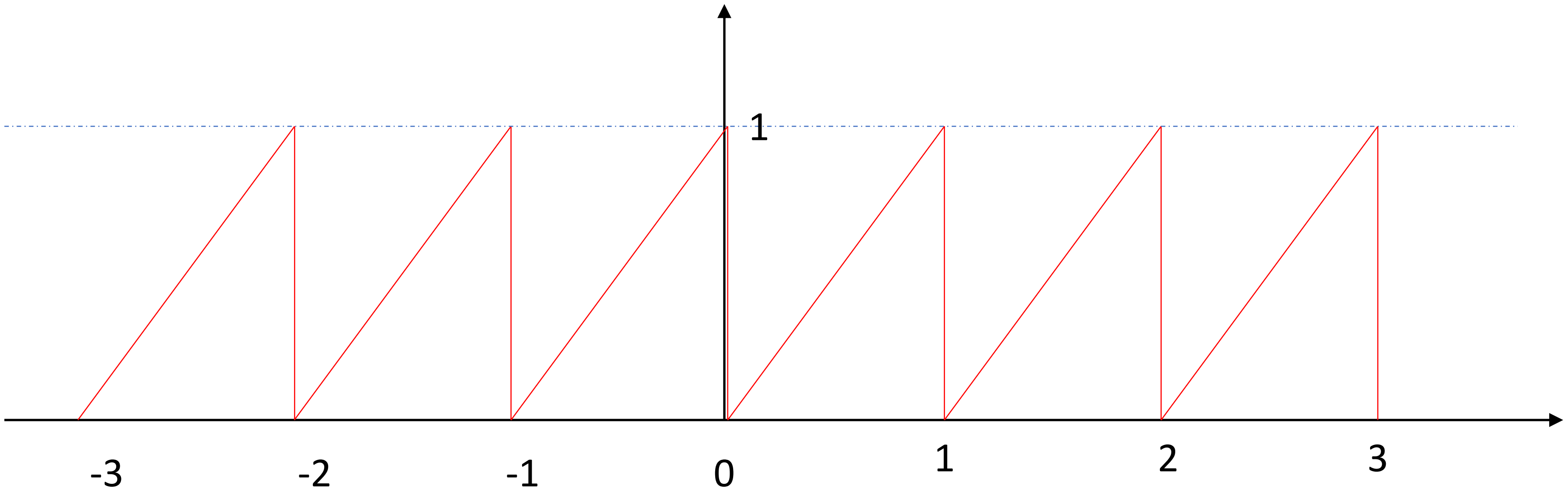
$$x(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{j\omega_o nt}$$

Where

$$C_n = \frac{1}{T}\int_{-\pi/2}^{\pi/2} x(t) e^{-j\omega_0 nt}$$

WORKED EXAMPLE

Find the exponential Fourier series of the sawtooth waveform shown below. Plot the magnitude and phase.



WORKED EXAMPLE / 01

Over one period, the signal may be represented as:

x(t) = t for $0 < t < 1$ and the period T = 1

The Fourier series coefficients of the signal x(t) are therefore

$C_n = \frac{1}{T}\int_0^T t\, e^{-i2\pi nt/T} dt = \int_0^1 te^{-i2\pi nt}\, dt$

Using the integration identity

$$\int xe^{ax}dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right)$$

Taking a = $-j2\pi n$ we get

$C_n = \frac{-j}{2\pi n}$ for $n \neq 0$

And

$$C_0 = \int_0^T x(t)dt = \int_0^1 tdt = \frac{1}{2}$$

WORKED EXAMPLE / 02

For $n \neq 0$

$$|C_n| = \sqrt{0 + \frac{1}{(2\pi n)^2}} = \frac{1}{2\pi n}$$

For n = 0

$$C_0 = \frac{1}{2}$$

The phase

$$\theta_n = \arg(C_n) = \pm 90^o$$