**EEEN 462 – ANALOGUE COMMUNICATION**

**FREQUENCY MODULATION (FM) - STUDY GUIDE/REVISION**

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## 1. INTRODUCTION TO FM

- **Definition:**
  Frequency Modulation (FM) varies the **frequency** of a carrier wave proportionally to the amplitude of the message signal. The **amplitude remains constant**.
- **Purpose:**
  Superior noise immunity compared to AM, ideal for high-fidelity audio (e.g., FM radio, audio broadcasting).
- **Key Components:**
  - **Carrier**: $c(t)=A_c\cos(2\pi f_c t)$
  - **Message Signal**: m(t) (e.g., audio, $f_m \ll f_c$).

## 2. MATHEMATICAL REPRESENTATION

- **Instantaneous Frequency**:

$$f_i(t) = f_c + k_f m(t)$$

  Where $k_f$ is the frequency sensitivity (Hz/volt).

- **FM Wave Equation**:

$$s(t) = A_c cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

**Phase Deviation**:

$$\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$$

## 3. MODULATION INDEX AND FREQUENCY DEVIATION

1. **Frequency Deviation ($\Delta f$)** is the peak shift from $f_c$ and is given by:

$$\Delta f = k_f |m(t)|_{max}$$

2. **Modulation Index ($\beta$)** indicates the extent to which the carrier frequency is varied by the modulating signal. It's calculated as the ratio of the frequency deviation (the maximum change in carrier frequency) to the modulating signal's frequency.

$$\beta = \frac{\Delta f}{f_m}$$

where

$\Delta f$ is the frequency deviation (the maximum change in carrier frequency)

$f_m$ is the maximum modulating signal's frequency

3. **Narrow Band FM (NBFM)** where β<1. NBFM is most suitable for applications where spectrum efficiency is crucial, such as two-way radio communication.

4. **Wideband FM (WBFM)** where β≫1. WBFM is used when high-quality signal transmission is prioritized, even at the expense of using more bandwidth.

## 4. SPECTRUM & BANDWIDTH

The frequency spectrum of a FM signal consists of a carrier frequency and multiple sidebands symmetrically distributed around it. The sidebands arise because FM modulation involves varying the carrier frequency in proportion to the modulating signal's amplitude, creating frequency components both above and below the carrier frequency.

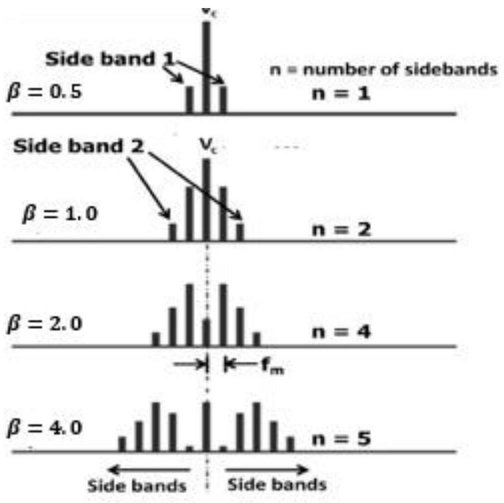


**Figure x.** Spectrum of FM modulated signal

- **Spectrum Characteristics**:

Infinite sidebands spaced at $f_m$ around $f_c$ (Bessel functions $J_n(\beta)$)

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\left[2\pi(f_c + nf_m)t\right]$$

- **Carson's Rule is** a formula used to estimate the bandwidth of a frequency modulated (FM) signal. It states that the bandwidth is approximately twice the sum of the maximum frequency deviation and the highest modulating frequency.

$$BW = 2(\Delta f + f_m)$$

$f_d \ll f_m$ which means $\beta \ll 1$

The bandwidth by the Carson's rule is given as

$$BW = 2f_m(1+\beta) \approx 2f_m$$

- **Example**:

Commercial FM radio transmitter has Δf=75 kHz and fm=15 kHz. Estimate the bandwidth of the FM signal?

$$BW = 2(\Delta f + f_m) = 2(75 + 15) = 2 \times 90 = 180 \text{ kHz}$$

## 5. POWER IN FM SIGNAL

- **Total Power of FM signal** is constant independent of $\beta$

$$P_t = \frac{A_c^2}{2}$$

- **Power in Carrier/Sidebands**:
  - Carrier power:

$$P_c = \frac{A_c^2}{2}[J_0(\beta)]^2$$

  - Sideband power:

$$P_{sb} = \frac{A_c^2}{2} \times \sum_{n \neq 0} [J_n(\beta)]^2$$

  - Power redistributes from carrier to sidebands as $\beta$ increases.

## 6. GENERATION OF FM WAVES

- **Direct Method**:
  - **Voltage-Controlled Oscillator (VCO)**: Output frequency varies with input voltage.

- **Indirect Method (Armstrong Method)**:

1. Generate NBFM using integrator + phase shifter.
2. Multiply frequency to achieve WBFM.

- **Varactor Diode Circuits**: Common in analogue FM generators.

## 7. DEMODULATION TECHNIQUES

- **Phase-Locked Loop (PLL)**:
  - Most widely used: VCO locks to input FM signal, error voltage = demodulated output.
- **Slope Detector**:
  - Convert FM to AM using an LC tank, then envelope detect.
  - Simple but distortion-prone.
- **Quadrature Detector**:
  - Mix FM signal with phase-shifted version; output proportional to frequency deviation.
  - Used in FM radio ICs.
- **Foster-Seeley Discriminator**:
  - Frequency-sensitive circuit producing linear voltage output.

## 8. DIFFERENCES BETWEEN AM & FM

| **AM** | **FM** |
|---|---|
| In Amplitude modulation, the amplitude of a carrier signal changed based on the data signal. AM radio broadcast signals utilize low-carrier frequencies to travel long distances. Sometimes, amplitude modulation signals are capable of bouncing off the ionosphere. As compared to FM, the distance travelled through the AM is high. | In frequency modulation, the carrier wave frequency can be changed based on the signal that holds data. The radio signals include high BW as compared to AM radio signals. These signals assist to provide good sound quality. FM also allows sending stereo signals. |
| In the mid-1870s, the first audio transmission was developed | FM was developed in the year 1930 in the US, by Edwin Armstrong. |
| In AM, the radio signal is known as a carrier signal & both the phase & frequency remain the same | In FM, the radio signal is known as a carrier signal, however, the amplitude, as well as phase, remain the same |
| Vulnerable to noise | Less vulnerable to noise |
| The sound clarity of AM is poor, however, can transmit long distances | FM has good sound quality |
| The AM broadcast frequency ranges from 535 kHz – 1705 kHz in medium frequency range. | The FM broadcast frequency ranges from 88 MHz – 108 MHz in VHF band |
| The modulation index of AM ranges from 0 to 1 | The modulation index of FM can be higher than 1 |

| AM | FM |
|---|---|
| It includes simply two sidebands | It includes a number of sidebands |
| It has an easy circuit | It has a difficult circuit |
| In AM, the carrier signal's amplitude can be changed to transmit the information. | In FM, the carrier signal's frequency can be changed to transmit the information |
| Broadcast AM has low bandwidth like 10 kHz. | Broadcast FM high bandwidth like 200 kHz |
| Broadcast AM operates in the MF (medium frequency) & HF( high frequency). | Broadcast FM works in the VHF band |

## 9. APPLICATIONS

1. **FM Radio Broadcasting** (88–108 MHz, Δf=75 kHz).
2. **TV Audio Transmission**.
3. **Two-Way Radios** (Police, emergency services).
4. **Medical Telemetry** (Patient monitoring).

## 10. PRACTICE PROBLEMS

1. **Problem**: An FM signal, m(t)=5cos(2π·1000t) has $f_c$=100 MHz and $k_f$=10 kHz/V. Find the frequency deviation Δf and the modulation index, β.

   **Solution**:

   $$\Delta f = k_f |m(t)|_{max} = 10 \times 5 = 50 \text{ kHz}$$

   $$\beta = \frac{\Delta f}{f_m} = \frac{50}{1} = 50$$

   $$\Delta f = k_f \cdot |m(t)|_{\max} = 10 \times 5 = 50 \, \text{kHz},$$
   $$\beta = \frac{\Delta f}{f_m} = \frac{50}{1} = 50.$$

2. **Problem**: Calculate bandwidth using Carson's rule for Δf=75 kHz, $f_m$=15 kHz

   **Solution**: BW=2(75+15)=180 kHz.

3. **Simulation**: Generate FM signals in Python for β=0.5$\beta$=0.5 (NBFM) and β=5$\beta$=5 (WBFM). Plot spectra.

```
python
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import spectrogram
```

```python
t = np.linspace(0, 1, 10000)

m_t = np.cos(2*np.pi*5*t)  # 5 Hz message

beta = 5

s_fm = np.cos(2*np.pi*100*t + beta*np.sin(2*np.pi*5*t))

f, t, Sxx = spectrogram(s_fm, fs=1000)

plt.pcolormesh(t, f, 10*np.log10(Sxx), shading='gouraud')

plt.ylabel('Frequency [Hz]'), plt.xlabel('Time [sec]')

plt.show()
```